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Stiffness Coefficients for Embedded Footings

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SYNOPSIS Mindlin's theory and Steinbrenner's approximation are used to evaluate the the static stiffness coefficients for embedded footings subjected to vertical oscillations. The footing is assumed to be embedded in an elastic layer underlain by a rigid base. The solution presented is simple and is amenable to hand calculations. It compares well with other rigorous solutions. In the analysis presented in the paper the shape of the footing can be easily taken into account. The method gives a solution which lies between the finite element solution and the rigorous closed form solutions.

INTRODUCTION

Several methods are available to evaluate the spring constants for discrete parameters, soil-structure systems subjected to dynamic vertical excitation of embedded footings (Kaldjian, 1969; Novak and Beredugo, 1972; Ramiah et al, 1977). All the methods assumed that the footing is resting on a semi-infinite mass. But there are instances where an elastic stratum is underlain by a rigid base. For such cases, Johnson et al (1975) have presented static finite element analysis describing the behaviour of embedded footing and considering all modes of displacement. The static stiffness coefficients may well be employed in a dynamic analysis involving soil-structure interaction under certain conditions. Generally, finite element solutions are complex and sophisticated and require a computer. Therefore, a simple method is proposed herein using the Mindlin theory and the Steinbrenner approximation for the settlement of an elastic layer underlain by a rigid base. The static stiffness coefficients are evaluated for a circular and square areas. The analysis is confined to the vertical mode only. The method could be extended to the sliding mode. The results of the present method are compared with that of the finite element solution. The advantage of this simplified method is that the shapes of the footing can be taken into account.

ANALYSIS

The vertical displacement due to a point load acting beneath the surface of a semi-infinite mass is given by the Mindlin theory. For an elastic stratum underlain by a rigid base, the displacement for an embedded footing could be calculated by integrating the Mindlin equation and using Steinbrenner's approximation (Terzaghi, 1943). Once the static displacement is known, the static stiffness coefficients can then easily be calculated.

a) Circular Footing: Integrating Mindlin's equation, the displacement below the centre of a circular embedded footing will be

$$w = \int_0^{2\pi}\int_0^{R} w_z \, dr \, d\theta \qquad \ldots (1)$$

in which R is the radius of the circular footing

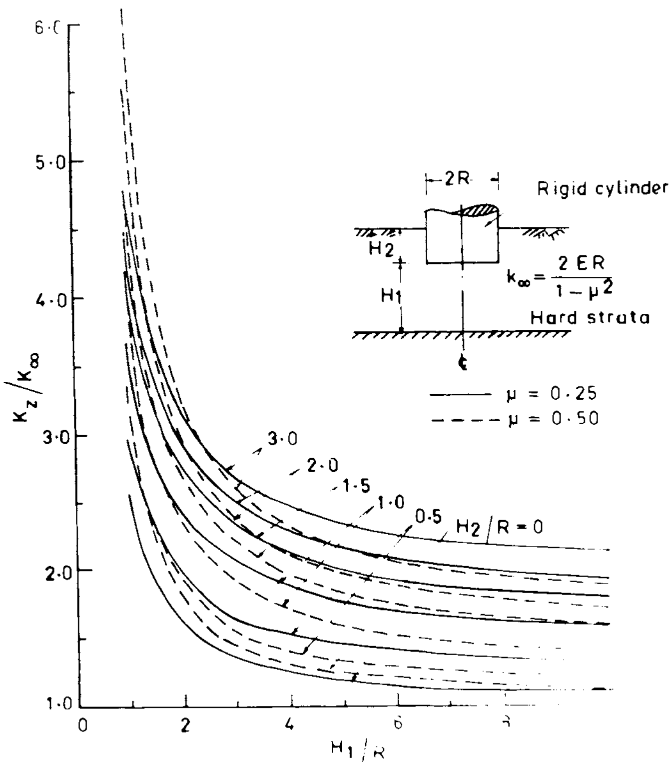

Fig. 1. Stiffness Coefficients for Circular Footing

and w_z is the displacement due to a point load as given by Mindlin's theory. The rigidity of the footing is taken into account by multiplying Eq.(1) by $\pi/4$. The displacement is first calculated as though the footing rests on an infinite mass (w_∞) and then the displacement at depth H_1 where hard strata exists (below the footing level) is calculated as w_{H1}. The displacement of the elastic layer is the taken as $w_e = w_\infty - w_{H1}$. The ratio of the total load to the displacement gives the stiffness coefficient. The results of the above method is presented in Fig. 1 for circular footing by plotting the dimensionless ratio k_z / k_∞

against H_1/R for $\mu = 0.25$ and 0.5. Here k_z represents the stiffness coefficient for embedded footing in an elastic layer and k_∞ is the stiffness coefficient of a circular footing resting on the surface of semi-infinite mass.

b) Rectangular Footing; Similar analysis is extended to rectangular area and the results are presented in Fig. 2 for square footing only ($a/b = 1$) where in the dimensionless ratio k_z/aE is plotted against H_1/a. Here 'E' is modulus of elasticity and 'a' is the least width of the footing. The results are obtained for $a/b = 2$ and 3 but not presented herein due to lack of space.

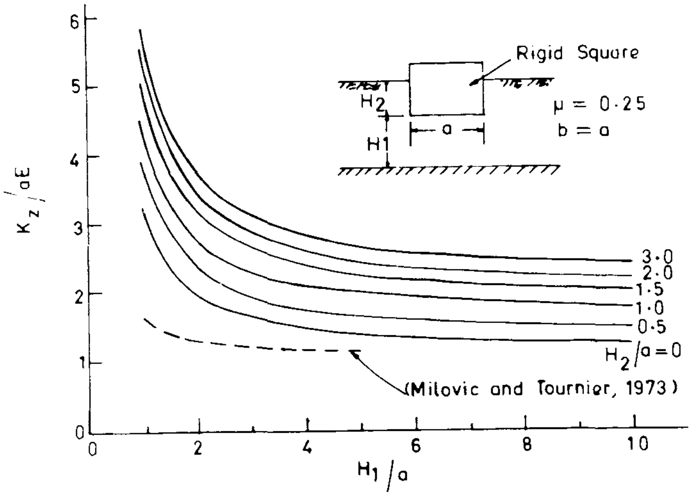

Fig. 2. Stiffness Coefficients for Square footing

DISCUSSION

(i) Circular footing: The results obtained from the above simplified method are compared with those of finite element solution (Johnson et al, 1975) for circular footing in Fig. 3. It is evident from the figure that the author's values are some what less than those obtained from the FEM and the difference between the two solution increases with the ratio H_2/R where H_2 is the depth of embedment. The maximum deviation of the author's solution from that of FEM is found over the range of $4 \leq H_1/R \leq 8$ and is about 15%. Here two points need to be mentioned. First, the Steinbrenner approximation underpredicts settlement to an extent of 10% resulting in overpredicting the stiffness by about the same amount. Secondly, there is a possibility that the finite element solution (Johnson et al, 1975) might have erred on the stiff side (Christiano, 1978). The analytical solution of Bycroft for the case of no embedment is also shown in Fig. 3 for comparison. There is close agreement between author's solution with that of Bycroft solution and the maximum difference between these solutionsis about 1%. Between Bycroft's solution and the FEM, there exist a difference of 11 to 17%.

(ii) Rectangular footing: Fig. 2 shows the variation of the ratio k_z/aE with H_1/a. It is seen that the ratio k_z/aE increases with H_2/a and decreases as H_1/a increases. Also shown in the figure is the Milovic and Tournier solution by the more rigorous method for surface footing i.e., $H_2/a = 0$. No finite element solution is availabe for rectangular or square footing. It is seen that the Milovic and Tournier solution gives lower values of the ratio k_z/aE and the difference between the two

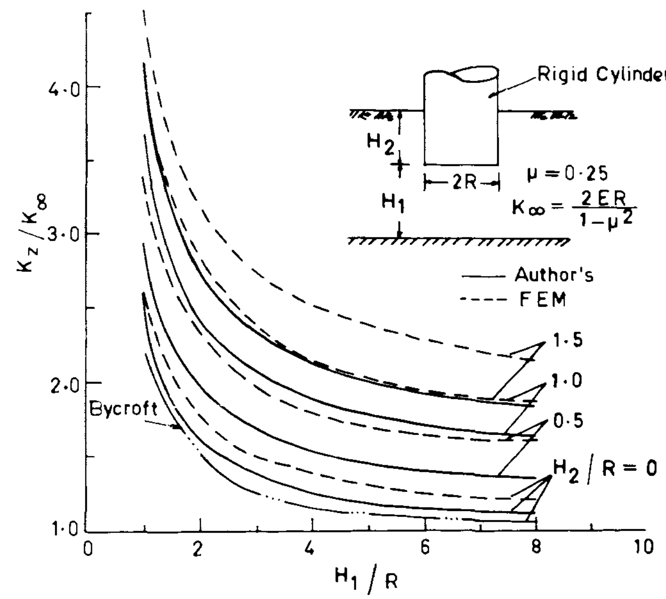

Fig. 3. Comparison with finite element solution

solutionsis 50% for $H_1/a = 1$. But the difference reduces to 17% for $H_1/a = 5$ and for the range $2 \leq H_1/a \leq 5$, the average difference is about 25%. Recognizing that the author's solution overpredicts the stiffness by 10%, the difference between the two solutions for the range $2 \leq H_1/a \leq 5$ reduces to 15% which appears to be reasonable considering the approximation made in the analysis.

CONCULSIONS

A simple method is proposed to evaluate the static stiffness coeffienets for an elastic layer underlain by a rigid base. The stiffness coefficients obtained by the method have been presented for a rigid circular and square footing embedded in an elastic stratum which rests on a rigid base. Results indicate that the depth of embedment of footing significantly affects the stiffness coefficients and the thickness of the elastic stratum is not very important. The stiffness coefficients obtained by the present method agrees fairly well with other solutions.

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